LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2024**

PMT1MC06 – PROBABILITY THEORY AND RANDOM PROCESSES

Date: 18-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

SECTION A – K1 (CO1)

	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	State Khintchin's theorem.
b)	Define multiple correlation coefficient.
c)	Write the sufficient conditions for consistent estimators.
d)	Describe the concept of critical region.
e)	Define Markov chain.

SECTION A – K2 (CO1)

	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	For a random variable X, which one is true? (i) $E(X^2)=[E(X)]^2$ (ii) $E(X^2)\le[E(X)]^2$ (iii) $E(X^2)\ge[E(X)]^2$ (iv) None
b)	If the regression lines coincide with each other, then the value of γ is (i) -1 (ii) +1 (iii) 0 (iv) ± 1
c)	The formula for the variance of a maximum likelihood estimator have been derived from: (i) Rao-Blackwell Theorem (ii) Cramer -Rao inequality (iii) LR test (iv) None
d)	Identify the symbol used for the power of the test: (i) α (ii) β (iii) $1-\alpha$ (iv) $1-\beta$
e)	The mean of the random process is often expressed as (i) Rank correlation (ii) Variance (iii) Ensemble average (iv) Auto-correlation

SECTION B – K3 (CO2)

	Answer any THREE of the following **(3 x 10 = 30)**
3	Let two random variables X and Y have the following joint probability density function: $f(x,y)=\begin{cases}\delta(2-x-y), 0\le x\le 1, 0\le y\le 1\\ 0, otherwise\end{cases}$. Find (i) the constant δ (ii) Marginal density functions of X and Y (iii) Conditional density functions (iv) Var (X), Var (Y) and (v) Cov (X,Y).
4	Using repeated rank method, calculate the rank correlation coefficient for the following data:

X	15	20	27	13	45	60	20	75
Y	50	30	55	30	25	10	30	70

5	Apply the Cauchy-Schwarz inequality and prove the Cramer-Rao inequality.

6	State and prove Neyman-Pearson fundamental lemma.
7	A class of modulated signals is modelled by the process $Y(t)=AX(t)\cos(\omega t+\theta)$, where $X(t)$ is the message signal which is a random process with mean 0 and autocorrelation function $R_{XX}(\tau)$. $A\cos(\omega t+\theta)$ is the carrier with amplitude A and the frequency ω are constants and the initial carrier phase θ is uniformly distributed in $[-\pi,\pi]$. Also $X(t)$ and θ are independent. Show that $Y(t)$ is a WSS process.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	State and prove weak law of large numbers. Also, show that the weak law of large numbers cannot be applied to the independent variables $X_1, X_2 \ldots X_n$ with equal probabilities $P[X_i=i]=\frac{1}{2}\wedge ¿$ $P[X_i=-i]=\frac{1}{2}, i=1,2 \ldots n$.
9	A random sample (X_1, X_2, X_3) of size 3 is drawn from a normal population with mean μ and variance σ^2. Let E_1, E_2 and E_3 are the estimators used to estimate mean value μ, where $E_1=X_1+X_2-X_3, E_2=2X_1+3X_3-4X_2$ and $E_3=\frac{1}{3}(\tau X_1+X_2+X_3)$. (i) Are E_1 and E_2 unbiased estimators? (ii) Calculate the value of τ such that E_3 is unbiased estimator for μ. (iii) Analyse the best estimator.
10	Given the frequency function $f(x,\theta)=\frac{1}{\theta}$, $0\le x\le\theta$ and 0 otherwise. Suppose you are testing the hypothesis $H_0:\theta=1$ against $H_1:\theta=2$ by means of a single observed value of x, then analyze the sizes of the type I and II errors if you choose the interval (i) $0.5\le x$ (ii) $1\le x\le 1.5$ as the critical regions?
11	Prove the following statements: (i) A first order stationary random process has a constant mean. (ii) For a second order stationary process, the autocorrelation function is a function of time difference. **(6+6.5)**
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	State and prove Chebyshev's inequality for continuous random variables. Further, calculate the upper bound for $P(\|X-2\|\ge 2)$, where the discrete variate X takes the value x with probability $2^{-x}, x=1,2,\ldots$.
13	Justify the following assertion using a mathematical proof. "The minimum variance unbiased estimator is always unique".
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
1	Create two medical-related data sets with exactly 12 data points each, and then explain how you can

4	utilize both the regression lines to predict unknown data.
15	Construct the three significant properties of random telegraph process and prove them.

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